Proposed problem to Octogon 2024

Let $a_k \ge a_{k+1} > 0$, $k = 1, 2 \dots$ Prove that if $\sum_{k=1}^{\infty} a_k$ diverges the series

$$\sum_{k=1}^{\infty} \left(\sum_{j=k}^{2k} \frac{a_j}{a_{j+1}}\right)^{-1} \text{ also diverges}$$